1 OVERVIEW

In our paper, we present asynchronous time integration to simulate fluids using a non-iterative EOS-based SPH solver. In this document, we provide additional images for comparison to previous methods (Sec. 2) and a quantitative velocity study of the time stepping methods (Sec. 3). Results are presented for fixed time stepping (FTS), globally adaptive time stepping (GTS) [Desbrun and Gascuel 1996], individual time stepping (ITS) [Ban et al. 2016], and our asynchronous time stepping (ATS).

Figure 1: Color map for the examples in this document.

In the following, velocities are color-coded using the color map in Fig. 1.
2 VISUAL COMPARISON TO FTS, GTS, AND ITS

2.1 Corner dam break

![Comparison of different time stepping methods for the corner dam break at different simulation times: there are only slight differences in the animations and we are able to achieve a speedup of a factor of 6.8 in parallel, and a factor of 7.3 in serial execution. Parameters for this scenario are given in the main paper.](image-url)

Figure 2: Side-by-side comparison of the different time stepping methods for the corner dam break at different simulation times: there are only slight differences in the animations and we are able to achieve a speedup of a factor of 6.8 in parallel, and a factor of 7.3 in serial execution. Parameters for this scenario are given in the main paper.
2.2 Radial flow (side view)

Figure 3: Side-by-side comparison of the different time stepping methods in the radial flow scenario at different simulation times: the overall flow looks similar for all tested integration methods. Using GTS, minor instabilities occur when particles hit the collision object, which can be observed at \( t_e = 3.5 \) s. We did not apply explicit shock handling and used the maximum possible values for the factors \( \lambda_v \) and \( \lambda_F \). There are also slight differences in the velocity field behind the collision object. At \( t_e = 3.5 \) s, we observe that our time integration method damps velocities at fluid interaction with the collision object. We also provide a top view of this scenario in Figure 4 to show this difference.
2.3 Radial flow (top view)

Figure 4: Top view of the radial flow scenario: we observe slight differences in the radial flow behind the collision object. Using our model, velocities are slightly damped compared to FTS at fluid interaction with the collision object. Using GTS or ITS, we also observe small differences compared to FTS, especially some instabilities resulting in isolated particles.
2.4 Fountain

Figure 5: Side-by-side comparison of the different methods in the fountain scenario at different simulation times: we observe noticeable differences in the overall shape of the fluid fountain. As mentioned in the paper, our model damps fluid velocities at interaction with collision objects. The velocities of the particles dropping on the top of the cone are damped in our model and the particles pouring down the fountain form a different shape compared to FTS, GTS, and ITS. Again, we observe instabilities using GTS. This results in many spraying particles, which can be observed at \( t_e = 2 \) s. Using ITS, we also recognize slight instabilities resulting in a large horizontal movement of the particles on top of the fountain. We use values twice as large as used by Ban et al. [2016] for \( \lambda_v \) and \( \lambda_F \).
3 QUANTITATIVE VELOCITY STUDY

In our paper, we focus on the performance aspects of our asynchronous time stepping model. Using the presented model, noticeable differences in the fluid flow can be observed as shown in the visual comparison in Sec. 5.2 of the main paper. To investigate these differences in detail a quantitative analysis is provided in this section.

The study consists of three parts: First, we inspect velocity differences between the time stepping models (Sec. 3.1) considered in the main document. Then, we provide measurements to evaluate velocity damping induced by the modification of the semi-implicit Euler (Sec. 3.2). As the time step size of an individual particle is governed by the CFL condition, we investigate velocities using different coefficients for \( \lambda_v \) (Sec. 3.3). Since we also consider forces acting on particle \( i \) [Ilmsen et al. 2010; Monaghan 1992], the influence of factor \( \lambda_F \) is included as well. We conduct this study comparing the mean as well as the 95th percentile velocities on the fountain scenario.

As mentioned in our paper, we observe differences in the fluid flow, in particular, at interaction with collision objects. In both investigated scenarios, only slight differences are noticeable when investigating the mean of the velocity magnitudes (Fig. 6). In the radial flow scenario, mean velocities are mainly governed by the radial force field. In the fountain scenario, they are quite small as most of the fluid is at rest. To reflect the observable variations (see the visual comparison) in the quantitative study, we exclude the parts of the simulation and investigate only the fluid velocities around the collision objects. Fig. 7 shows the regions considered in this study.

3.1 Model comparison

In the radial flow scenario, we observe differences in the velocity field by visual inspection, in particular, behind the bunny (see Sec. 2.2). As expected, we observe from Fig. 8 (left), that the mean velocities are smaller using ATS compared to FTS, and are slightly larger when using GTS or ITS. Regarding the 95th percentile (Fig. 8, right), the same characteristics are present.

![Figure 6: Mean of the velocity magnitudes for all particles over time using different time stepping models. In the radial flow scenario (left), we recognize only slight differences. Using our time stepping model, velocities are slightly smaller compared to FTS. Using GTS and ITS, velocities are larger compared to FTS. Regarding the fountain scenario (right), the overall velocity is quite small. Using ATS, larger values occur compared to the other time stepping models, but more importantly, oscillations occur if using ITS. They result in instabilities in the simulation and the fluid starts to vibrate at the start of the simulation (see accompanying video).](image)

![Figure 7: Clipping boxes for isolated investigations of the fluid velocities around the collision objects. On the left, a top view of the radial flow scenario is shown. The considered region is outlined by a yellow rectangle. On the right, a side view of the fountain scenario including the considered region is shown.](image)
Figure 8: Velocity measurements of the radial \(/f_{\text{low}}\) scenario on the selected region as shown in Fig. 7. The mean velocities are smaller using our time stepping model, but the overall behavior is quite similar. Regarding the 95th percentile, there are also only slight differences.

Figure 9: Velocity measurements of the fountain scenario. Mean and 95th percentile of all particles are shown on the left and for the selected region (Fig. 7) on the right. Regarding the mean and the 95th percentile of the selected region, we observe smaller velocities using our time stepping model (ATS) compared to FTS, GTS, and ITS, which show almost similar behavior.

Regarding the fountain scenario, differences in the fluid behavior become more apparent. As mentioned in Sec. 2.4, we recognize noticeable variances in the overall fluid flow using the considered time stepping models. They result in distinct overall shapes of the fluid fountain. As expected, this fact can also be observed in a quantitative study of the particle velocities. Fig. 9 illustrates measurements of the velocity magnitudes. On the left, the mean and the 95th percentile is depicted. We observe that the mean velocity is slightly larger when using our method compared to FTS, GTS and ITS. Another important fact is revealed as well: using ITS, the mean velocities show large oscillations. This can also be observed by visual inspection as the resting fluid beneath the fountain starts to vibrate at the beginning of the simulation. At \(t = 4\) s, the fluid hits the top of the collision object and bounces off the structure. In the 95th percentile plot considering all particles (Fig. 9, second from left), this is reflected in a rapid change of the velocity magnitudes at this point when using FTS, GTS, or ITS. With our time stepping model, particle velocities are damped and almost no change is observable. Neglecting the resting fluid beneath the fountain (Fig. 9, first and second from right), this effect becomes more prominent when using our time stepping model. However, it is less noticeable compared to the other investigated time stepping models. This complies with the visual inspection: due to the damped velocities, the fluid particles have a steeper drop down from the collision object using our model.

3.2 Comparison of the semi-implicit Euler and the used modification

As we use a modification of the semi-implicit Euler step that may introduce additional numerical damping, we investigate its influence on the velocity field by comparing it to a simulation conducted using the unmodified semi-implicit Euler step. We measure the velocities similar to the study conducted in Sec. 3.1. We restrict our investigations in this study to GTS and ATS, and present the measurements for the radial flow as well as for the fountain scenario. In Fig. 10, results of this study are plotted. We cannot observe major differences between the
Figure 10: Comparison of the used integrator and the semi-implicit Euler. Velocity measurements are given for the radial flow scenario using GTS and ATS. From left to right: mean and 95th percentile of all particles, and mean and 95th percentile of selected particles around the collision object (Fig. 7). There are no major differences in the velocity plots using the different time integration methods. Using GTS, there are no important differences regarding the mean and the 95th percentile. With ATS, there are only slight differences, which are negligible.

Figure 11: Comparison of the used integrator and the semi-implicit Euler. Velocity measurements are given for the fountain scenario using GTS and ATS. From left to right: Mean and 95th percentile of all particles, and mean and 95th percentile of selected particles around the collision object (Fig. 7). Using GTS, we do not observe any important differences. With ATS, different mean and 95th velocities occur if considering all particles. If neglecting the fluid beneath the fountain, these variations decrease. In particular, there is no velocity damping caused by the integration scheme. Nonetheless, we were able to use slightly larger values for $\lambda_v$ and $\lambda_F$ using this integration scheme and, as we focus on performance, we prefer to use the modification to the semi-implicit Euler.

3.3 Influence of the step size
We found in the previous studies that our time stepping model leads to more damping of velocities compared to FTS, GTS, and ITS, especially when interacting with collision objects. In this section, we study the influence of the time step size on this effect by varying the values of the coefficients $\lambda_v$ and $\lambda_F$ for the CFL condition. As these differences become most apparent in the fountain scenario, we present measurements for this scenario (Fig. 12).
We are able to recognize some effects: If neglecting the resting fluid, we do not recognize any differences in the mean velocities nor in the 95th percentile. When including all particles into the measurements, larger fluid velocities occur when increasing the factor $\lambda_F$. This is caused by larger movements of the almost resting particles beneath the fountain. We can conclude that we do not observe any velocity damping due to the time step size and, furthermore, we assume that the time step size of a particle is mostly restricted by the force term of the CFL condition.

REFERENCES


